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Generalised supersymmetric fluxbranes

José Figueroa-O'Farrill

Department of Mathematics and Statistics, The University of Edinburgh
Scotland, United Kingdom
E-mail: j.m.figueroa@ed.ac.uk

Joan Simón

The Weizmann Institute of Physical Sciences, Department of Particle Physics
Rehovot, Israel
*E-mail:*jsimon@weizmann.ac.il

Abstract: We classify generalised supersymmetric fluxbranes in type II string theory obtained as Kaluza-Klein reductions of the Minkowski space vacuum of eleven-dimensional supergravity. We obtain two families of smooth solutions which contains all the known solutions, new solutions called nullbranes, and solutions interpolating between them. We explicitly construct all the solutions and we study the U-duality orbits of some of these backgrounds.

Keywords: M-Theory, p-branes, Differential and Algebraic Geometry.

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Contents

1. Introduction and conclusions

There has recently been a lot of interest in the embedding of the Melvin universe [1] in M/string theory [2], in particular in the type IIA flux 7-brane (F7-brane), whose M-theory description [3]–[6] strongly suggests that type IIA string theory with magnetic field M_s^2/g_s^2 is dual to type 0A string theory [7, 8]. They also play an important role in the supergravity description of the expansion of a Dp-brane into a spherical D$(p+2)$-brane due to the dielectric effect [9] and in the possible stabilisation of tubular branes [10, 11]. The study of analogous magnetic backgrounds allowing a conformal field theory description [12, 13, 14] provides a framework to study the decay of unstable backgrounds into stable supersymmetric ones since they are smoothly connected to the supersymmetric closed string vacuum. Related work can also be found in [15]–[20].

It was pointed out in [6, 21] that when several magnetic parameters are taken into account the corresponding supergravity background may preserve some amount of supersymmetry. One of the goals of this paper is to classify these possibilities, or equivalently, to classify supersymmetric fluxbranes in string theory. To do so, we shall reexamine the geometrical setting giving rise to the flux-fivebrane (or F5-brane, for short) [6] in the context of Kaluza-Klein reductions. Starting with eleven-dimensional Minkowski space, the F5-brane is obtained by considering a Kaluza-Klein reduction along the orbits generated by a Killing vector consisting of a translation and a rotation. The reduction will preserve some supersymmetry if the rotation belongs to the isotropy algebra of some nonzero Killing spinor of the Minkowski vacuum. This suggests considering the Kaluza-Klein reduction by the most general Killing vector in Minkowski space subject to some natural conditions, namely that the quotient be a smooth lorentzian manifold. This problem has been addressed in [5], although there no particular attention has been paid to supersymmetric fluxbranes or to Lorentz transformations other than spatial rotations. The space of Killing vectors is of course isomorphic to the Poincaré algebra, and the reductions satisfying these conditions form a subset of this algebra. Generic points in this parameter space will break supersymmetry completely, but there are special loci for which supersymmetry is preserved. In this paper we determine these loci. There are two families of solutions intersecting in a common two-parameter family.

Any translation preserves supersymmetry, but a Lorentz transformation which does must belong to the isotropy algebra of some spinor. In dimension greater than two, there are three types of (pure) Lorentz transformations: boosts, rotations and null rotations. Only null rotations and some "special" rotations preserve spinors. Reductions involving these special rotations generate type IIA configurations which can be interpreted as flux p-branes (or Fp-branes, for short) for $p = 1, 3, 5$, but we also have the possibility of reducing along null rotations. This gives rise to configurations which we call *nullbranes* (or Np-branes) as they have a null RR 2-form field strength. Whereas supersymmetric fluxbranes can be interpreted as intersections of F7-branes, nullbranes can be thought of infinitely boosted F7-branes,[1] even though their physical interpretation is not clear to us at the present time, these being time dependent backgrounds in string theory preserving one half of the spacetime supersymmetry. Reductions along linear combinations of the Killing vectors giving rise to the flux/null-branes are also possible. The type IIA configurations obtained in this way interpolate continuously between the fluxbranes and nullbranes discussed before.

The formulation developed in this paper allows an immediate extension to many other supersymmetric M-theory backgrounds. This is discussed and applied to M-brane backgrounds and their intersections in a separate article [22], in which we construct supersymmetric composite configurations of type IIA/B branes (D-branes, fundamental strings, NS5-branes, waves, etc) and flux/null-branes. In a second forthcoming article we consider supersymmetric reductions of the maximally supersymmetric M-theory backgrounds of AdS type [23].

There remain many interesting open questions regarding F-branes such as which D-brane configurations are possible in these backgrounds (or their U-duals) and which is the corresponding field theory one can define on them in certain decoupling limits. Furthermore, while standard D-branes admit an open string description on flat space, the existence of such a description for F-branes remains an open question.

[1] JS would like to thank M. Berkooz for suggesting this interpretation.

This paper is organised as follows. In section 2 we discuss the geometric context of the present paper: namely the supersymmetric Kaluza-Klein reduction of the flat M-theory vacuum and we classify those reductions resulting in supersymmetric smooth IIA backgrounds, what we call (smooth, generalised) supersymmetric IIA fluxbranes. We will see that there are two families of generalised supersymmetric fluxbranes: one family contains the supersymmetric fluxbranes recently discussed in [6, 21, 24] and in addition a novel $\frac{1}{4}$-BPS fluxstring solution, and the other family contains new solutions called here null-branes as well as new solutions which interpolate between them and some of the known fluxbranes. We can summarise the possible generalised supersymmetric fluxbranes as follows. As elementary objects, we have a $\frac{1}{2}$-BPS F5-brane, a $\frac{1}{2}$-BPS N7-brane, a $\frac{1}{4}$-BPS F3-brane, a $\frac{1}{4}$-BPS F1-brane and an $\frac{1}{8}$-BPS F1-brane. We also find interpolating solutions (which we call generalised fluxbranes): a $\frac{1}{4}$-BPS solution interpolating between the F5- and the N7-branes and a $\frac{1}{8}$-BPS solution interpolating between the F3- and the N7-branes. All these geometries are constructed explicitly in section 3. Finally section 4 contains a comprehensive analysis of the U-dual configurations which can be obtained from the solutions constructed in section 3. This is illustrated with the aid of diagrams showing the U-duality orbits of some of the elementary fluxbranes.

Note added. There is some overlap between the results in this paper and those in [24], which appeared as we were completing the present work.

2. Kaluza-Klein reduction and generalised fluxbranes

We start by considering Kaluza-Klein reductions of the maximally supersymmetric vacuum of M-theory described by eleven-dimensional Minkowski space, and in this way classify all the smooth supersymmetric solutions of type IIA supergravity which are Kaluza-Klein reductions of the M-theory vacuum. These geometries will be described explicitly in the next section.

2.1 The geometrical setting

The general geometrical setting is the following (see also [5]). We will consider a one-parameter subgroup of the group of isometries of Minkowski space, in other words a one-dimensional subgroup G of the Poincaré group acting on Minkowski space. Topologically, G is either a circle or a line. In traditional treatments of Kaluza-Klein reduction, G is always taken to be a circle subgroup; but circles do not act freely on Minkowski space. It is possible to follow tradition and demand that G be a circle; but in order to have a smooth quotient, one is forced to introduce "identifications" in Minkowski space which effectively close the integral curves of the Killing vector generating the action. This practice is standard in the context of fluxbranes, and although it has its merits, we prefer not to follow it here. To restate, all our solutions will be smooth quotients of Minkowski space by the action of $G \cong \mathbb{R}$. If desired, these solutions *could* be viewed as Kaluza-Klein reductions on a circle. Indeed, quotienting by $\mathbb{R}$ can be done in two steps: quotienting by $\mathbb{Z}$ (i.e., making identifications in Minkowski space) and then quotienting by the circle $\mathbb{R}/\mathbb{Z}$. The action of $\mathbb{Z}$ introduces a length scale in the problem, which is (2π times) the radius of the circle.

If the G-action is free and has spacelike orbits,[2] the ten-dimensional space obtained by Kaluza-Klein reduction will be a solution of the type IIA supergravity. In addition, this solution will be supersymmetric provided that G leaves some spinor invariant. Let ξ be the Killing vector generating the G action infinitesimally. The action will be (locally) free if and only if ξ is nowhere-vanishing. With our choice of (mostly plus) metric, demanding that the norm $\|\xi\|^2$ be everywhere positive guarantees that the orbits are spacelike. Under further mild restrictions (namely that every point in Minkowski space should have trivial stabiliser) the space of orbits will be a smooth manifold inheriting a lorentzian metric.

In adapted coordinates, where $\xi = \partial_z$, we can write the eleven-dimensional Minkowski metric as

$$ds^2(\mathbb{E}^{10,1}) = e^{-2\phi/3} g + e^{4\phi/3}(dz + A)^2\,, \tag{2.1}$$

where g is the ten-dimensional metric in the string frame, ϕ is the dilaton and A is the RR 1-form potential of type IIA supergravity. By construction, the triple (g, ϕ, A) will satisfy the equations of motion of IIA supergravity, provided that we set the other field strengths F_3 and F_4 in the theory to zero. This solution will be supersymmetric if and only if ξ preserves some Killing spinors. In the case where we make identifications in Minkowski space in order to view this as a circle reduction, it is convenient to introduce a length scale R, the radius of the circle, and then write $z = R\chi$, where χ is an angular variable taking values in $\mathbb{R}/2\pi\mathbb{Z}$. In this case, it is also convenient to rescale the RR 1-form potential, so that the metric becomes the more familiar

$$ds^2(\mathbb{E}^{10,1}) = e^{-2\phi/3} g + e^{4\phi/3} R^2 \left(d\chi + C_{(1)}\right)^2\,. \tag{2.2}$$

The Killing vector ξ acts on a Killing spinor ε via the spinorial Lie derivative (see, e.g., [26] and also [27]), defined by

$$L_\xi \varepsilon = \nabla_\xi \varepsilon + \frac{1}{4}\nabla_a \xi_b \Gamma^{ab}\varepsilon\,. \tag{2.3}$$

Since Killing spinors are parallel in the Minkowski vacuum, the condition that ξ preserves supersymmetry becomes the algebraic condition that, for some nonzero parallel spinor ε,

$$\frac{1}{4}\nabla_a \xi_b \Gamma^{ab}\varepsilon = 0\,. \tag{2.4}$$

Alternatively, relative to adapted coordinates, a spinor is invariant under $\xi = \partial_z$ if and only if it does not depend explicitly on z.

The most general Killing vector in Minkowski space is the sum of a translation and a Lorentz transformation:

$$\xi = a^\mu \partial_\mu + \frac{1}{2}\omega^{\mu\nu}(x_\mu \partial_\nu - x_\nu \partial_\mu)\,, \tag{2.5}$$

[2] Since we want to induce a metric in the space of orbits, it is important that the Killing vector never be null; hence it has to be always spacelike or always timelike. Since we are interested in constructing solutions of type II supergravity in signature (9,1) we restrict ourselves to spacelike orbits. Nothing prevents us from considering timelike orbits and in this way obtain fluxbrane solutions of euclidean supergravity theories of the type considered in [25], but this will be left for another time.

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where $\omega^{\mu\nu} = -\omega^{\nu\mu}$, and the condition (2.4) guaranteeing the preservation of supersymmetry becomes

$$\frac{1}{4}\omega^{\mu\nu}\Sigma_{\mu\nu}\varepsilon = 0\,. \tag{2.6}$$

In other words, we see that the translation can be arbitrary and that the Lorentz transformation must belong to the isotropy algebra of a spinor.

For the purposes of this paper, by a *(generalised) fluxbrane* we will mean a smooth solution of IIA supergravity obtained as the Kaluza-Klein reduction of eleven-dimensional Minkowski space. In what follows we will classify these and construct two multiparameter families of generalised supersymmetric fluxbranes containing as special cases all the supersymmetric fluxbranes which have been hitherto considered in the literature.

2.2 Classification of smooth supersymmetric fluxbranes

Let us outline the mathematical problem of classifying supersymmetric generalised fluxbranes (of type IIA supergravity). This problem reduces to finding free actions of a group $G \cong \mathbb{R}$ on Minkowski space preserving the metric, leaving some parallel spinors invariant and possessing spacelike orbits. In other words, we are interested in classifying one-parameter subgroups of the spinor isotropy groups of (the spin cover of) the Poincaré group, which act freely on Minkowski space with spacelike orbits, up to conjugation.

It is more convenient to work in terms of the Lie algebra. Killing vectors in Minkowski space are in one-to-one correspondence with the Poincaré algebra $\mathfrak{p} = \mathfrak{l} \ltimes \mathfrak{t} \cong \mathfrak{so}(10,1) \ltimes \mathbb{R}^{10,1}$, where $\mathfrak{l}$ is the Lorentz subalgebra and $\mathfrak{t}$ is the translation ideal. Let $\mathfrak{m} \subset \mathfrak{p}$ denote the *subset* corresponding to Killing vectors ξ which obey the following conditions

1. $\|\xi\|^2 = g(\xi,\xi) > 0$; and

2. ξ preserves some spinors.

Notice that if $\xi \in \mathfrak{m}$ then $s\xi \in \mathfrak{m}$ for any nonzero real number s. Although there is physics in the scale s, it is convenient for classification purposes to identify any two collinear Killing vectors, as they have the same orbits, but parametrised differently. We therefore introduce the *moduli space* $\mathcal{M}$ of supersymmetric Kaluza-Klein reductions of flat space as the real projective space of the subset $\mathfrak{m}$. We now proceed to determine $\mathcal{M}$.

Let ξ be a Killing vector and let us write it uniquely as

$$\xi = \tau + \lambda\,,$$

where $\tau \in \mathfrak{t}$ is a translation and $\lambda \in \mathfrak{l}$ is a Lorentz transformation. The translation component cannot be zero, since every Lorentz transformation fixes a point (the "origin") and hence vanishes there. Moreover, since $\|\xi\|^2 = \|\tau\|^2$ at the origin, condition (1) above says that τ must be spacelike at the origin, and hence everywhere. Our strategy to determine the moduli space $\mathcal{M}$ will be the following: we will find a normal form for ξ exploiting the freedom to conjugate by the Poincaré group, and then impose the conditions on the normal form.

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By conjugating with a Lorentz transformation we can bring λ to a normal form. Which form depends on the type of element it is. A generic $\lambda \in \mathfrak{so}(10,1)$ fixes one direction. This direction can be spacelike, timelike or null. Accordingly, λ can take one of the following three normal forms:

(spacelike)

$$\lambda = B_{01}(\gamma) + R_{23}(\beta_1) + R_{45}(\beta_2) + R_{67}(\beta_3) + R_{89}(\beta_4)\,; \tag{2.7}$$

(timelike)

$$\lambda = R_{12}(\beta_1) + R_{34}(\beta_2) + R_{56}(\beta_3) + R_{78}(\beta_4) + R_{9\natural}(\beta_5)\,; \tag{2.8}$$

(null)

$$\lambda = N_{+1}(u) + R_{23}(\beta_1) + R_{45}(\beta_2) + R_{67}(\beta_3) + R_{89}(\beta_4)\,, \tag{2.9}$$

where $\pm = \natural \pm 0$ with $\natural$ the tenth spacelike direction, $R_{ij}(\beta)$ is an infinitesimal rotation with parameter β in the (ij) plane, $B_{0i}(\gamma)$ is an infinitesimal boost with parameter γ along the ith direction and $N_{+i}(u)$ is a null rotation with parameter u in the ith direction.

Bringing λ to one of these normal forms does not use up all the freedom of conjugation, since we can still conjugate by those elements of the Poincaré group which fix the normal form. All normal forms are fixed under conjugation by translations, which corresponds to changing the origin in Minkowski space. It is possible to change the origin in order the bring the translation τ to normal form. This normal form depends on λ. Under a change of origin,

$$\xi = \tau + \lambda \mapsto \tau + [\lambda, \tau'] + \lambda\,,$$

whence we can, by choosing the origin appropriately, get rid of any component of τ in the image of $[\lambda, -]$.

If λ has normal form (2.7), by a change of origin and a rescaling (which we are allowed to do since the moduli space $\mathcal{M}$ is projective) we can always choose $\tau = \partial_\natural$. In particular this means that as vector fields in Minkowski space, τ and λ are orthogonal, whence

$$\|\xi\|^2 = \|\tau\|^2 + \|\lambda\|^2\,.$$

It is easy to see that if the boost parameter $\gamma \neq 0$ then there are points in Minkowski space where ξ is not spacelike: simply take $|x^1|$ large enough. Therefore for spacelike orbits, we require $\gamma = 0$, whence λ also fixes a timelike direction, so it is a special case of the normal form (2.8).

If λ has normal form (2.8) and all β's are different from zero, there exists a choice of origin in which τ is timelike, violating one of the above conditions. Therefore at most four β's can be nonzero and hence there exist coordinates in which, after rescaling, $\xi = \partial_\natural + \lambda$, where λ takes the form

$$\lambda = R_{12}(\beta_1) + R_{34}(\beta_2) + R_{56}(\beta_3) + R_{78}(\beta_4) \in \mathfrak{so}(8)\,.$$

Now, supersymmetry never constrains the translation component, but it does impose constraints on λ. Here $\lambda \in \mathfrak{so}(8)$ preserves a spinor if and only if it belongs to a $\mathfrak{spin}(7)$

subalgebra. By permuting the coordinates if necessary, this condition translates into the vanishing of the sum of the β's. This gives rise to a three-parameter family of vector fields

$$\xi = \partial_\natural + R_{12}(\beta_1) + R_{34}(\beta_2) + R_{56}(\beta_3) + R_{78}(\beta_4)\,, \tag{2.10}$$

with $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 0$.

Finally if λ has normal form (2.9), we must again have some parameter among u, β_i vanishing, for otherwise by a suitable change of origin we could set $\tau = a\partial_-$, violating the condition that τ be spacelike. Therefore either u or one of the β's must vanish. If $u = 0$ we are back in the case treated previously. Therefore let us assume that $u \neq 0$ and that $\beta_4 = 0$, say. By choosing the origin suitably we can write $\xi = \tau + \lambda$, where

$$\tau = a\partial_- + b\partial_8 + c\partial_9 \quad \text{and} \quad \lambda = N_{+1}(u) + R_{23}(\beta_1) + R_{45}(\beta_2) + R_{67}(\beta_3)\,.$$

The condition that τ be spacelike forces either b or c to be nonzero. Moreover the condition that ξ be spacelike forces $a = 0$. To see this simply notice that

$$\begin{aligned} \|\xi\|^2 &= \|\tau\|^2 + \|\lambda\|^2 + 2g(\tau,\lambda) \\ &= \|\tau\|^2 + \|\lambda\|^2 - 2aux^1\,, \end{aligned} \tag{2.11}$$

where $\|\tau\|^2 > 0$, $\|\lambda\|^2 \geq 0$ are independent of x^1 and hence if we take $|x^1|$ large enough we eventually find that $\|\xi\|^2$ is not positive. Therefore in some coordinates, and after rescaling,

$$\xi = \partial_9 + N_{+1}(u) + R_{23}(\beta_1) + R_{45}(\beta_2) + R_{67}(\beta_3)\,.$$

It remains to impose that ξ preserves some spinors. This means that for some nonzero spinor ε, its Lie derivative $L_\xi\varepsilon$ along ξ vanishes. Using (2.6) this is equivalent to the algebraic equation

$$(u\Sigma_{+1} + \beta_1\Sigma_{23} + \beta_2\Sigma_{45} + \beta_3\Sigma_{67})\,\varepsilon = 0\,.$$

To solve it, notice that this equation is of the form

$$(N+S)\varepsilon = 0\,, \tag{2.12}$$

where $N = u\Sigma_{+1}$ is nilpotent (in fact, $N^2 = 0$) and $S = \beta_1\Sigma_{23} + \beta_2\Sigma_{45} + \beta_3\Sigma_{67}$ is semisimple and moreover N and S commute. As a result, (2.12) is true if and only if

$$N\varepsilon = 0 \qquad \text{and} \qquad S\varepsilon = 0\,.$$

The first equation says that $\Gamma_+\varepsilon = 0$ and the second equation says that $S \in \mathfrak{so}(6)$ is actually in an $\mathfrak{su}(3)$ subalgebra. Again, up to a permutation of the coordinates, this means that the sum of the β's vanishes.

We can summarise the above results as follows. There are two families of spacelike Killing vectors in Minkowski space which induce supersymmetric Kaluza-Klein reductions. There exists a coordinate system $(z, y^i, y^\pm)$, with $i = 1, \ldots, 8$, where the metric takes the form

$$ds^2(\mathbb{E}^{10,1}) = 2dy^+dy^- + \sum_{i=1}^{8} dy^i dy^i + dz^2 \tag{2.13}$$

and where, to an overall scale, the two families of Killing vectors are given by

$$\xi = \partial_z + R_{12}(\beta_1) + R_{34}(\beta_2) + R_{56}(\beta_3) + R_{78}(\beta_4)\,, \qquad \sum_i \beta_i = 0\,, \tag{2.14}$$

and by

$$\xi = \partial_z + N_{+1}(u) + R_{34}(\beta_1') + R_{56}(\beta_2') + R_{78}(\beta_3')\,, \qquad \sum_i \beta_i' = 0\,. \tag{2.15}$$

We should point out that the parameter u is ineffective; that is, it is only important to distinguish between two cases $u = 0$ and $u \neq 0$. In this latter case, we can set u to any desired nonzero value by rescaling $y^{\pm} \mapsto c^{\pm 1} y^{\pm}$ for a suitable c. Such rescalings are of course Lorentz transformations.

To complete the proof that these vector fields give rise to smooth reductions, we must ensure that the integrated action is free: meaning that no point is left fixed and every point has trivial stabiliser. Indeed, the action on Minkowski space of the element $\exp t\xi$ in the subgroup $G \cong \mathbb{R}$ generated by ξ takes the form

$$(z, y^{\mu}) \mapsto (z + t, \Lambda(t)^{\mu}{}_{\nu} y^{\nu})\,, \tag{2.16}$$

whence the action is manifestly free. This means that the space of orbits $\mathbb{E}^{10,1}/G$ is smooth. Let us now describe the explicit geometry of these supersymmetric fluxbranes.

3. Generalised supersymmetric fluxbranes

To best describe the explicit geometry of the generalised supersymmetric fluxbranes obtained by reducing Minkowski space along the orbits of the Killing vectors given by equations (2.14) and (2.15), we will work in coordinates adapted to the Killing vector. This turns out to be very easy, once we observe that ξ is simply a dressed version of its translation component:

$$\xi = U \partial_z U^{-1} \qquad \text{where} \quad U = \exp(-z\lambda)\,. \tag{3.1}$$

Let us introduce coordinates $(x^i, x^{\pm})$ related to the coordinates $(y^i, y^{\pm})$ by

$$x = U\,y\,. \tag{3.2}$$

It follows easily that $\xi x = 0$, so that x are good coordinates for the space of orbits. Since λ is an infinitesimal Lorentz transformation, the x are linearly related to the y with z-dependent coefficients. Indeed, let us denote by B the constant 10×10 real matrix such that

$$\lambda y = By\,. \tag{3.3}$$

Therefore the new coordinates are given by

$$x = \exp(-zB)\,y\,, \tag{3.4}$$

where exp here means the matrix exponential. For λ in either of the normal forms described at the end of the last section, it is very easy to write an explicit expression for this matrix exponential; but we will refrain from doing so here, as it is not needed in order to write down the IIA background.

Indeed, it is a simple matter to rewrite the metric (2.13) in terms of the new variables, obtaining

$$ds^2(\mathbb{E}^{10,1}) = \Lambda(dz + A)^2 + dx^\dagger \left(1 - \Lambda^{-1}(Bx)(Bx)^\dagger\right) dx \,, \tag{3.5}$$

where $v^\dagger = v^t\eta$ is the adjoint relative to the Minkowski metric, and

$$\Lambda = 1 + (Bx)^\dagger(Bx) \qquad \text{and} \qquad A = \Lambda^{-1}(Bx)^\dagger dx \,. \tag{3.6}$$

Using the Kaluza-Klein ansatz (2.1) we can read off the IIA background which has $F_3 = F_4 = 0$ and in the string frame the nontrivial fields are given by

$$\phi = \tfrac{3}{4}\log\Lambda \qquad \text{and} \qquad g = \Lambda^{1/2} dx^\dagger \left(1 - \Lambda^{-1}(Bx)(Bx)^\dagger\right) dx \,, \tag{3.7}$$

together with A in the previous equation, which also contains the definition of Λ. The parameters in this solution are hidden in B, which is the matrix of an infinitesimal Lorentz transformation of the form described in equations (2.14) or (2.15). As discussed in the previous section, we can always choose the x coordinates in such a way that B only depends on 3 real parameters which are unconstrained; although in one of the cases the parameter is ineffective. We keep it because we can in this way discuss the limit $u \to 0$. Some subvarieties of the moduli space of supersymmetric fluxbranes are already known, as we now discuss.

To help comparison with the literature, let us write down the matrix B explicitly relative to the basis $\{x^i, x^+, x^-\}$:

$$B = \begin{pmatrix} 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u \\ \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 & 0 \\ -u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.8}$$

where either $u = 0$ or $\beta_1 = 0$ (or both) and where $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 0$. We will now consider several special cases.

3.1 Supersymmetric fluxbranes

If $u = 0$, so that the Lorentz transformation is purely a rotation, we obtain, in addition to the standard supersymmetric fluxbranes considered previously in [6, 21, 24], a novel fluxstring configuration preserving 1/4 of the supersymmetry. The solutions described here correspond to the supersymmetric points in the space of solutions described in [5]; although that work did not address the question of supersymmetry.

We have several possibilities depending on how many of the β's are zero. If three β's vanish, then supersymmetry implies that all β's must vanish and the resulting configuration is the type IIA vacuum. Therefore, if we are to obtain a nontrivial supersymmetric fluxbrane, at most two β's can be zero.

3.1.1 The supersymmetric F5-brane revisited

If $\beta_1 = \beta_2 = 0$, and hence $\beta_3 = -\beta_4 = \beta$, we recover the supersymmetric F5-brane of [6]. The matrix B corresponds to an element in the Cartan subalgebra of $\mathfrak{sp}(1) \subset \mathfrak{so}(4) \subset \mathfrak{so}(9,1)$:

$$B = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&-\beta&0&0&0&0 \\ 0&0&0&0&\beta&0&0&0&0&0 \\ 0&0&0&0&0&0&0&\beta&0&0 \\ 0&0&0&0&0&0&-\beta&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \tag{3.9}$$

Here β is the parameter related to the magnetic field in [6]. The solution is given explicitly by (3.7) with

$$\Lambda = 1 + \beta^2 r^2 \quad \text{and} \quad A = \frac{\beta}{1+\beta^2 r^2}\left(x^5 dx^6 - x^6 dx^5 - x^7 dx^8 + x^8 dx^7\right), \tag{3.10}$$

where r is the radial coordinate in the 4-plane spanned by the coordinates $x^5, \ldots, x^8$ transverse to the fluxbrane. We can rewrite this solution in a more familiar form, by introducing coordinates:

$$x^5 + ix^6 = r\cos\theta e^{i(\psi+\varphi)} \qquad \text{and} \qquad x^7 + ix^8 = r\sin\theta e^{i(\psi-\varphi)}, \tag{3.11}$$

where $\theta \in [0, \pi/2]$, $\psi \in [0, \pi]$ and $\varphi \in \mathbb{R}/2\pi\mathbb{Z}$. In terms of the new coordinates,

$$A = \frac{\beta r^2}{1+\beta^2 r^2}\left(d\varphi + \cos(2\theta) d\psi\right), \tag{3.12}$$

and the metric becomes

$$g = \Lambda^{1/2}\left[ds^2(\mathbb{E}^{5,1}) + dr^2 + r^2 d\theta^2 + r^2\sin^2(2\theta) d\psi^2\right] + +\Lambda^{-1/2} r^2 \left(d\varphi + \cos 2\theta d\psi\right)^2 . \tag{3.13}$$

This solution represents a F5-brane. Indeed, the IIA RR 2-form field $F = dA$ in this solution has a nontrivial "charge", as can be seen from the (normalised) integral of $F \wedge F$ on the transverse $\mathbb{E}^4$:

$$\frac{1}{8\pi^2}\int_{\mathbb{E}^4} F\wedge F = \lim_{\rho\to\infty}\frac{1}{8\pi^2}\int_{r\leq\rho} F\wedge F = \lim_{\rho\to\infty}\frac{1}{8\pi^2}\int_{r=\rho} A\wedge F = \frac{1}{\beta^2}, \tag{3.14}$$

where we have used that the orientation on the 3-sphere $r = \rho$ induced by the natural orientation $dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8$ on $\mathbb{R}^4$ agrees with $d\theta \wedge d\varphi \wedge d\psi$. The F5-brane preserves one half of the supersymmetry of the eleven-dimensional vacuum.

The near-horizon geometry ($r \to 0$) of the F5-brane is flat. Indeed the metric is asymptotic to $\mathbb{E}^{5,1}$ times a cone metric:

$$dr^2 + r^2 \left[d\theta^2 + \sin^2(2\theta) d\psi^2 + (d\varphi + \cos(2\theta) d\psi)^2\right] . \tag{3.15}$$

The metric of the base of the cone (that is, the quantity in square brackets) is that of the round 3-sphere, whence the cone metric is that of $\mathbb{E}^4$. In the other limit ($r \to \infty$) we obtain a conformally cylindrical geometry. Indeed, h is asymptotically conformal to a metric which is a product of $\mathbb{E}^{5,1}$ with

$$dr^2 + r^2 \left[d\theta^2 + \sin^2(2\theta) d\psi^2\right] + \frac{1}{\beta^2}(d\varphi + \cos(2\theta) d\psi)^2 . \tag{3.16}$$

This is the metric on the total space of a circle bundle over $\mathbb{R}^3$ where the radius of the circle is constant and equal to $1/\beta$.

3.1.2 The supersymmetric F3-brane

If $\beta_1 = 0$, but the rest of the β's are different from zero (but add up to zero) we obtain the supersymmetric flux-threebrane discussed in [21, 24]. In this case the matrix B is a Cartan subalgebra of $\mathfrak{su}(3) \subset \mathfrak{so}(6) \subset \mathfrak{so}(9,1)$:

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.17}$$

The solution is again given explicitly by (3.7) with

$$\Lambda = 1 + \sum_{i=2}^{4} \beta_i^2 |z_i|^2 \quad \text{and} \quad A = \frac{\Lambda^{-1}}{2i} \sum_{i=2}^{4} \beta_i \left(\bar{z}_i dz_i - z_i d\bar{z}_i\right) , \tag{3.18}$$

where z_i, $i = 2, 3, 4$, correspond to complex coordinates in the 6-plane spanned by the real coordinates $x^3, \ldots, x^8$ transverse to the fluxbrane. We can rewrite this in terms of the radial distance to the fluxbrane by introducing coordinates:

$$\begin{aligned} x^3 + ix^4 &= z_1 = r \cos\theta_1 e^{i\varphi_1} \\ x^5 + ix^6 &= z_2 = r \sin\theta_1 \cos\theta_2 e^{i\varphi_2} \\ x^7 + ix^8 &= z_3 = r \sin\theta_1 \sin\theta_2 e^{i\varphi_3} , \end{aligned} \tag{3.19}$$

where $\theta_{1,2} \in [0, \pi/2]$ and $\varphi_i \in \mathbb{R}/2\pi\mathbb{Z}$ for all i. In terms of the new coordinates, the scalar function becomes

$$\Lambda = 1 + r^2 \left(\beta_2^2 \cos^2\theta_1 + \beta_3^2 \sin^2\theta_1 \cos^2\theta_2 + \beta_4^2 \sin^2\theta_1 \sin^2\theta_2\right), \tag{3.20}$$

whereas the RR 1-form potential is given by

$$A = \Lambda^{-1} r^2 \left(\beta_2 \cos^2\theta_1 d\varphi_1 + \beta_3 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \beta_4 \sin^2\theta_1 \sin^2\theta_2 d\varphi_3\right), \tag{3.21}$$

and the metric becomes

$$\begin{aligned} g = \Lambda^{1/2} \Big[& ds^2(\mathbb{E}^{3,1}) + dr^2 + r^2\left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2\right) + \\ & + r^2\left(\cos^2\theta_1 d\varphi_1^2 + \sin^2\theta_1\cos^2\theta_2 d\varphi_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\varphi_3^2\right)\Big] - \\ & - \Lambda^{-1/2} r^4 \left(\beta_2 \cos^2\theta_1 d\varphi_1 + \beta_3 \sin^2\theta_1\cos^2\theta_2 d\varphi_2 + \beta_4 \sin^2\theta_1 \sin^2\theta_2 d\varphi_3\right)^2 . \end{aligned} \tag{3.22}$$

The solution preserves $1/4$ of the supersymmetry.

3.1.3 Supersymmetric fluxstrings

If none of the β's vanish, there are two possibilities. Either the β's add up to zero pairwise or they do not. In the former case, the matrix B belongs to a Cartan subalgebra of $\mathfrak{sp}(1) \oplus \mathfrak{sp}(1) \subset \mathfrak{so}(4) \oplus \mathfrak{so}(4) \subset \mathfrak{so}(8) \subset \mathfrak{so}(9,1)$, whereas in the latter, it belongs to a Cartan subalgebra of $\mathfrak{su}(4) \subset \mathfrak{so}(8) \subset \mathfrak{so}(9,1)$. In either case we have a supersymmetric fluxstring. In the former case the fluxstring preserves $1/4$ of the supersymmetry, whereas in the latter case it preserves $1/8$. The latter case was discussed in [21, 24], the former case seems new.

Let us examine, first of all, the case when B belongs to a Cartan subalgebra of $\mathfrak{sp}(1) \oplus \mathfrak{sp}(1) \subset \mathfrak{so}(4) \oplus \mathfrak{so}(4) \subset \mathfrak{so}(8) \subset \mathfrak{so}(9,1)$:

$$B = \begin{pmatrix} 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tilde\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde\beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\tilde\beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.23}$$

It is natural to use the same coordinates introduced in (3.11), this time to parametrise the 4-plane spanned by $x^1, \ldots, x^4$

$$x^1 + ix^2 = r\cos\theta e^{i(\psi+\varphi)} \qquad \text{and} \qquad x^3 + ix^4 = r\sin\theta e^{i(\psi-\varphi)}, \tag{3.24}$$

where $\theta \in [0, \pi/2]$, $\psi \in [0, \pi]$ and $\varphi \in \mathbb{R}/2\pi\mathbb{Z}$, and proceed analogously with the second 4-plane spanned by $x^5, \ldots, x^8$

$$x^5 + ix^6 = \tilde{r} \cos\tilde{\theta} e^{i(\tilde{\psi}+\tilde{\varphi})} \qquad \text{and} \qquad x^7 + ix^8 = \tilde{r} \sin\tilde{\theta} e^{i(\tilde{\psi}-\tilde{\varphi})}\,, \tag{3.25}$$

where $\tilde{\theta} \in [0, \pi/2]$, $\tilde{\psi} \in [0, \pi]$ and $\tilde{\varphi} \in \mathbb{R}/2\pi\mathbb{Z}$. The full distance to the fluxstring is measured by the square root of $r^2 + \tilde{r}^2$. In terms of the new coordinates, the scalar function becomes

$$\Lambda = 1 + r^2\beta^2 + \tilde{r}^2\tilde{\beta}^2 \tag{3.26}$$

fully determining the dilaton, $\phi = (3/4) \log \Lambda$, whereas the RR 1-form potential is given by

$$A = \Lambda^{-1}\beta r^2 \left(d\varphi + \cos(2\theta) d\psi\right) + \Lambda^{-1}\tilde{\beta}\tilde{r}^2 \left(d\tilde{\varphi} + \cos(2\tilde{\theta}) d\tilde{\psi}\right), \tag{3.27}$$

and the metric becomes

$$\begin{aligned} g = \Lambda^{1/2} \Big[& ds^2(\mathbb{E}^{1,1}) + dr^2 + r^2 d\theta^2 + r^2 \sin^2(2\theta) d\psi^2 + \\ & + d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2 + \tilde{r}^2 \sin^2(2\tilde{\theta}) d\tilde{\psi}^2 \Big] + \\ + \Lambda^{-1/2} \Big[& r^2 \left(d\varphi + \cos 2\theta d\psi\right)^2 (1 + \tilde{\beta}\tilde{r}^2) + \tilde{r}^2 \left(d\tilde{\varphi} + \cos 2\tilde{\theta} d\tilde{\psi}\right)^2 (1 + \beta r^2) - \\ & - 2\beta\tilde{\beta} r^2 \tilde{r}^2 \left(d\varphi + \cos 2\theta d\psi\right) \left(d\tilde{\varphi} + \cos 2\tilde{\theta} d\tilde{\psi}\right) \Big]\,. \end{aligned} \tag{3.28}$$

Finally, let us consider the case in which B belongs to a Cartan subalgebra of $\mathfrak{su}(4) \subset \mathfrak{so}(8) \subset \mathfrak{so}(9,1)$:

$$B = \begin{pmatrix} 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.29}$$

The solution is again given explicitly by (3.7) with

$$\Lambda = 1 + \sum_{i=1}^{4} \beta_i^2 |z_i|^2 \quad \text{and} \quad A = \frac{\Lambda^{-1}}{2i} \sum_{i=1}^{4} \beta_i \left(\bar{z}_i dz_i - z_i d\bar{z}_i\right), \tag{3.30}$$

where z_i $i = 1, 2, 3, 4$ correspond to complex coordinates in the 8-plane spanned by the real coordinates $x^1, \ldots, x^8$ transverse to the fluxbrane. We can rewrite this in terms of the radial distance to the fluxbrane by introducing coordinates:

$$\begin{aligned}
x^1 + ix^2 &= z_1 = r\cos\theta_1 e^{i\varphi_1} \\
x^3 + ix^4 &= z_2 = r\sin\theta_1\cos\theta_2 e^{i\varphi_2} \\
x^5 + ix^6 &= z_3 = r\sin\theta_1\sin\theta_2\cos\theta_3 e^{i\varphi_3} \\
x^7 + ix^8 &= z_4 = r\sin\theta_1\sin\theta_2\sin\theta_3 e^{i\varphi_4} \,,
\end{aligned} \tag{3.31}$$

where $\theta_i \in [0, \pi/2]$ and $\varphi_i \in \mathbb{R}/2\pi\mathbb{Z}$ for all $i = 1, 2, 3, 4$. In terms of the new coordinates, the scalar function becomes

$$\begin{aligned}
\Lambda = 1 + r^2 \Big(&\beta_1^2 \cos^2\theta_1 + \beta_2^2 \sin^2\theta_1 \cos^2\theta_2 + \\
&+ \beta_3^2 \sin^2\theta_1 \sin^2\theta_2 \cos^2\theta_3 + \beta_4^2 \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 \Big) \,.
\end{aligned} \tag{3.32}$$

The latter fixes the dilaton to be $\phi = \frac{3}{4}\log\Lambda$, whereas the RR 1-form potential is given by

$$\begin{aligned}
A = \Lambda^{-1} r^2 \Big(&\beta_1 \cos^2\theta_1 d\varphi_1 + \beta_2 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \\
&+ \beta_3 \sin^2\theta_1 \sin^2\theta_2 \cos^2\theta_3 d\varphi_3 + \beta_4 \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 d\varphi_4 \Big) \,,
\end{aligned} \tag{3.33}$$

and the metric by

$$\begin{aligned}
g = \Lambda^{1/2} \Big[&ds^2(\mathbb{E}^{1,1}) + dr^2 + r^2 \left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 \right) + \\
&+ r^2 \Big(\cos^2\theta_1 d\varphi_1^2 + \sin^2\theta_1 \cos^2\theta_2 d\varphi_2^2 + \sin^2\theta_1 \sin^2\theta_2 \cos^2\theta_3 d\varphi_3^2 + \\
&+ \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 d\varphi_4^2 \Big) \Big] - \\
-\Lambda^{-1/2} r^4 \Big[&\beta_1 \cos^2\theta_1 d\varphi_1 + \beta_2 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \beta_3 \sin^2\theta_1 \sin^2\theta_2 \cos^2\theta_3 d\varphi_3 + \\
&+ \beta_4 \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 d\varphi_4 \Big]^2 \,.
\end{aligned} \tag{3.34}$$

3.2 Supersymmetric nullbranes

On the other extreme, we have these reductions where u is different from zero, but the β's are zero. These reductions give rise to solutions with null RR field strengths which we tentatively call *nullbranes*. In this case we have the freedom to choose a coordinate system in which the null rotation is along one of the coordinates, say x^1. The corresponding matrix B has all β's equal to zero and only u is different from zero. Moreover, it is possible to reabsorb u, provided it is nonzero, by rescaling $x^{\pm} \mapsto u^{\pm 1} x^{\pm}$. The resulting solution seems to be new.[3] Explicitly, we have

$$\Lambda = 1 + (x^-)^2 \quad \text{and} \quad A = \frac{1}{1+(x^-)^2}\left(x^- dx^1 - x^1 dx^-\right), \tag{3.35}$$

a non-trivial dilaton $\phi = \frac{3}{4}\log\Lambda$ and a type IIA metric in the string frame given by

$$g = \Lambda^{1/2}\left[2dx^+ dx^- - (x^1)^2 (dx^-)^2 + ds^2\left(\mathbb{E}^7\right)\right] + +\Lambda^{-1/2}\left(dx^1 + x^1 x^- dx^-\right)^2 . \tag{3.36}$$

The RR 2-form field strength is null

$$F = \frac{2}{\Lambda^2} dx^- \wedge dx^1 , \tag{3.37}$$

hence our name for these solutions. These solutions always preserve one half of the supersymmetry.

Notice that the metric (3.36) has an $\mathbb{E}^7$ subspace, so this suggests computing the Hodge dual of the null field strength (3.37). Indeed, $\star F$ couples naturally to a seven-dimensional extended object. Doing so one finds that $\star F$ is actually constant in this coordinate system and is given by

$$\star F = 2dx^- \wedge \mathrm{dvol}(\mathbb{E}^7) \tag{3.38}$$

for our choice of orientation. This is reminiscent of the homogeneous branes (H-branes) introduced in [28] (see also [29]).

3.3 Generalised supersymmetric fluxbranes

When the matrix B contains both rotations and null rotations, the resulting solution interpolates between the fluxbranes and the nullbranes discussed above. To illustrate the geometry of these solutions, let us consider a matrix B which is the sum of the matrix (3.9) giving rise to the F5-brane and the matrix giving rise to the N7-brane:

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & 0 & 0 \\ -u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \tag{3.39}$$

As in the case of the nullbrane, one can reabsorb u by a rescaling $x^{\pm} \mapsto u^{\pm 1} x^{\pm}$. Applying the general formulae (3.6) and (3.7), it is easy to write down the resulting IIA background.

[3] The possibility of quotienting by boosts was mentioned briefly in [6]. The resulting fluxbranes are electrically charged but are not supersymmetric.

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In the coordinates (3.11), the dilaton is again given by $\phi = (3/4)\log\Lambda$, the RR 1-form potential by

$$A = \Lambda^{-1}\left(x^- dx^1 - x^1 dx^- + \beta r^2 (d\varphi + \cos 2\theta d\psi)\right) \tag{3.40}$$

and the metric by

$$\begin{aligned} g = \Lambda^{1/2}\left[2dx^+dx^- - (x^1)^2(dx^-)^2 + ds^2(\mathbb{E}^3) + dr^2 + r^2(d\theta^2 + \sin^2 2\theta d\psi^2)\right] + \\ +\Lambda^{-1/2}\Big[\left(dx^1 + x^1x^-dx^-\right)^2 + r^2 (d\varphi + \cos 2\theta d\psi)^2 \left(1 + (x^-)^2\right) + \\ + (\beta r)^2 \left((dx^1)^2 + (x^1)^2(dx^-)^2\right) - 2\beta r^2 (x^-dx^1 - x^1dx^-)(d\varphi + \cos 2\theta d\psi)\Big] \,, \end{aligned}$$

where the scalar function is $\Lambda = 1 + (x^-)^2 + \beta^2 r^2$.

This solution smoothly interpolates between the F5-brane — which is recovered in the region $x^- \ll 1$ keeping βr fixed — and the N7-brane, in the region $\beta r \ll 1$ keeping x^- fixed. It preserves 1/4 of the spacetime supersymmetry.

There is one more class of solutions which interpolates between the N7-brane and the F3-brane. In this case the matrix B is the sum of the matrix (3.17) and the matrix giving rise to the N7-brane

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 & 0 \\ -u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \tag{3.41}$$

The solution has three parameters since $\beta_2 + \beta_3 + \beta_4 = 0$, but as argued before, one can reabsorb u by a rescaling $x^\pm \mapsto u^{\pm 1} x^\pm$. The type IIA configuration can be written down by proceeding as before; applying the general formulae (3.6) and (3.7), and this time using the coordinates (3.19). The dilaton is given by $\phi = (3/4)\log\Lambda$, the RR 1-form potential by

$$\begin{aligned} A = \Lambda^{-1}\Bigg(x^-dx^1 - x^1dx^- + r^2 \times \\ \times \left(\beta_2 \cos^2\theta_1 d\varphi_1 + \beta_3 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \beta_4 \sin^2\theta_1 \sin^2\theta_2 d\varphi_3\right)\Bigg) \end{aligned}$$

and the metric by

$$
\begin{aligned}
g = \Lambda^{1/2} \Big[& 2dx^+dx^- + (dx^1)^2 + (dx^2)^2 + +dr^2 + r^2 \left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2\right) + \\
& + r^2 \left(\cos^2\theta_1 d\varphi_1^2 + \sin^2\theta_1 \cos^2\theta_2 d\varphi_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\varphi_3^2\right) \Big] - \\
-\Lambda^{-1/2} \Big[& r^4 \left(\beta_2 \cos^2\theta_1 d\varphi_1 + \beta_3 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \beta_4 \sin^2\theta_1 \sin^2\theta_2 d\varphi_3\right)^2 + \\
& + \left(x^- dx^1 - x^1 dx^-\right)^2 + 2r^2 \left(x^- dx^1 - x^1 dx^-\right) \cdot \\
& \cdot \left(\beta_2 \cos^2\theta_1 d\varphi_1 + \beta_3 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \beta_4 \sin^2\theta_1 \sin^2\theta_2 d\varphi_3\right) \Big] \,, \qquad (3.42)
\end{aligned}
$$

where the scalar function is

$$
\Lambda = 1 + (x^-)^2 + r^2 \left(\beta_2^2 \cos^2\theta_1 + \beta_3^2 \sin^2\theta_1 \cos^2\theta_2 + \beta_4^2 \sin^2\theta_1 \sin^2\theta_2\right).
$$

The above configuration preserves 1/8 of the supersymmetry of the eleven-dimensional vacuum. It smoothly interpolates between the F3-brane — which is recovered in the region $x^- \ll 1$ keeping $\beta_i\, r$ fixed for all i — and the N7-brane, in the region $\beta_i\, r \ll 1\ \forall\, i$ keeping x^- fixed. Notice that in both interpolating solutions presented in this subsection, as we move in the parameter space we find regions of supersymmetry enhancement, corresponding to the N7-brane and the F5/F3-branes themselves.

4. Dualities and flux branes

Any supergravity background gives rise to a family of related solutions through the use of U-duality transformations and other solution-generating techniques. The purpose of this section is to discuss the family of solutions obtained by U-duality from some of the generalised fluxbranes found in the previous section, namely fluxbranes and nullbranes. It is well known that any time a T-duality transformation is used to generate a new solution, the latter is delocalised in the T-dual direction. It is often possible to find localised versions of the new solutions, but it remains to be seen whether this is indeed true for the solutions we find here. We will make much use of the T-duality rules derived by Bergshoeff, Hull and Ortín in [30].

4.1 A word on the notation

In applying the T-duality rules to the solutions found above, we will obtain many solutions for which no name yet exists. In this section we have introduced a notation in order to be able to identify them. Since the notation is not standard, let us take a moment to explain it. By an F(p, q)-brane, we shall denote a solution with full Poincaré invariance in $p + 1$ dimensions, but which is nevertheless delocalised in q of them. A similar notation, now $N(p, q)$ is employed in the section on nullbrane dualities. Whenever an S-duality transformation is applied, an s is added to the notation together with one of the letters a or b to emphasise that this is a configuration of type IIA or IIB, respectively. This is done to distinguish solutions which, although formally equivalent, belong to different theories. In the section on fluxstring dualities, we further adorn the notation with a subscript 1 or 2, to distinguish between the two fluxstring configurations. The rest of the notation is standard.

4.2 F5-brane dualities

The starting solution is the F5-brane, with metric (3.13) and RR 1-form potential (3.12), whereas the dilaton is given by $\phi = \frac{3}{4}\log(1+\beta^2 r^2)$.

By performing a T-duality along $5-p$ worldspace directions of the F5-brane, one obtains a family of $\mathrm{F}(p,5-p)$-brane configurations, for $p=4,3,2,1,0$, belonging to type IIA for odd p and to type IIB for even p. They can be jointly described by

$$\begin{aligned} \phi &= \frac{p-2}{4}\log\Lambda\,, \qquad G_{(7-p)} = F\wedge \mathrm{dvol}(\mathbb{E}^{5-p}) \\ g &= \Lambda^{1/2}ds^2(\mathbb{E}^{p,1}) + \Lambda^{-1/2}ds^2(\mathbb{E}^{5-p}) + \Lambda^{1/2}h\,, \end{aligned} \tag{4.1}$$

where the transverse metric h is defined as

$$h = dr^2 + r^2(d\theta^2 + \sin^2 2\theta d\psi^2) + \frac{r^2}{\Lambda}(d\varphi + \cos 2\theta d\psi)^2\,, \tag{4.2}$$

and $G_{(7-p)}$ stands for the RR field strength, F being the RR 2-form field strength of the original F5-brane configuration.

Notice that the $\mathrm{F}(2,3)$-brane is S-selfdual, whereas the $\mathrm{F}(4,1)$-brane is S-dual to the $\mathrm{F}(4,1)bs$-brane

$$\begin{aligned} \phi &= -\frac{1}{2}\log\Lambda\,, \qquad H = -F\wedge dx \\ g &= ds^2(\mathbb{E}^{4,1}) + \Lambda^{-1}dx^2 + h\,, \end{aligned} \tag{4.3}$$

where H is the NS-NS three-form field strength. Similarly, the $\mathrm{F}(0,5)$-brane is S-dual to the $\mathrm{F}(0,5)bs$-brane

$$\begin{aligned} \phi &= \frac{1}{2}\log\Lambda\,, \qquad \star H = F\wedge \mathrm{dvol}(\mathbb{E}^5) \\ g &= \Lambda[-dt^2 + h] + ds^2(\mathbb{E}^5)\,. \end{aligned} \tag{4.4}$$

By a T-duality along $\mathbb{R}^4$ in the $\mathrm{F}(4,1)bs$-brane, we obtain the $\mathrm{F}(4,1)$ as-brane, which is formally equivalent to its T-dual. Both are T-duals along the twisted directions to the corresponding vacuum solutions in type IIA/B with non-trivial identifications. Similarly, the new $\mathrm{F}(0,5)bs$ -brane configuration also generates through T-duality a new type IIA solution, the $\mathrm{F}(0,5)as$-brane, which is again equivalent to its type IIB T-dual.

Once new type IIA configurations have been generated, they can also be understood as the Kaluza-Klein reductions of certain new M-theory configurations. In particular, the $\mathrm{F}(4,1)as$-brane and the $\mathrm{F}(3,2)$-brane are the reductions of the $\mathrm{F}(4,2)$-brane

$$\begin{aligned} F_{(4)} &= F\wedge \mathrm{dvol}(\mathbb{E}^2) \\ ds^2_{11} &= \Lambda^{1/3}[ds^2(\mathbb{E}^{4,1}) + h] + \Lambda^{-2/3}ds^2(\mathbb{E}^2)\,, \end{aligned} \tag{4.5}$$

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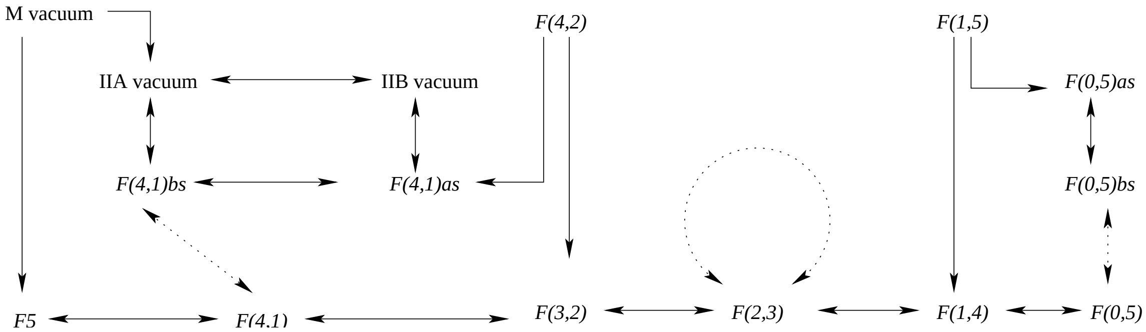

Figure 1: Web of dualities associated with an F5-brane. A dotted line represents an S-duality transformation, a unidirectional solid line represents a Kaluza-Klein reduction from M-theory, and a bidirectional solid line a T-duality transformation.

whereas the F$(1,4)$-brane and the F$(0,5)as$ -brane come from the F$(1,5)$-brane

$$\begin{aligned} \star F_{(4)} &= F \wedge \mathrm{dvol}(\mathbb{E}^5) \\ ds^2_{11} &= \Lambda^{2/3}[-ds^2(\mathbb{E}^{1,1}) + h] + \Lambda^{-1/3} ds^2(\mathbb{E}^5)\,. \end{aligned} \tag{4.6}$$

This is illustrated in figure 1.

4.3 F3-brane dualities

The starting point is the F3-brane, with metric (3.22) and RR 1-form potential (3.21), whereas the dilaton is given by $\phi = (3/4)\log\Lambda$, Λ being the one defined in (3.20).

Proceeding as in the previous subsection, one can generate a family of F$(p,3-p)$-brane configurations by performing T-duality transformations along $3-p$ of the the worldspace directions of the original F3-brane. They belong to type IIA for odd p and to type IIB for even p. They are jointly described by

$$\begin{aligned} \phi &= \frac{p}{4}\log\Lambda\ , \qquad G_{(5-p)} = F \wedge \mathrm{dvol}(\mathbb{E}^{3-p}) \\ g &= \Lambda^{1/2} ds^2(\mathbb{E}^{p,1}) + \Lambda^{-1/2} ds^2(\mathbb{E}^{3-p}) + h\,, \end{aligned} \tag{4.7}$$

where the transverse metric h is defined by

$$\begin{aligned} h = {} & \Lambda^{1/2}\Big[dr^2 + r^2\left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2\right) + \\ & + r^2\left(\cos^2\theta_1 d\varphi_1^2 + \sin^2\theta_1\cos^2\theta_2 d\varphi_2^2 + \sin^2\theta_1\sin^2\theta_2 d\varphi_3^2\right)\Big] - \\ & -\Lambda^{-1/2} r^4\left(\beta_2\cos^2\theta_1 d\varphi_1 + \beta_3\sin^2\theta_1\cos^2\theta_2 d\varphi_2 + \beta_4\sin^2\theta_1\sin^2\theta_2 d\varphi_3\right)^2\ . \end{aligned} \tag{4.8}$$

Notice that the F$(0,3)$-brane is S-selfdual — in particular, it has a constant dilaton — whereas the F$(2,1)$-brane is S-dual to the F$(2,1)bs$-brane described by

$$\begin{aligned} \phi &= -\frac{1}{2}\log\Lambda\ , \qquad H = -F \wedge dx \\ g &= ds^2(\mathbb{E}^{2,1}) + \Lambda^{-1} dx^2 + \Lambda^{-1/2} h\,. \end{aligned} \tag{4.9}$$

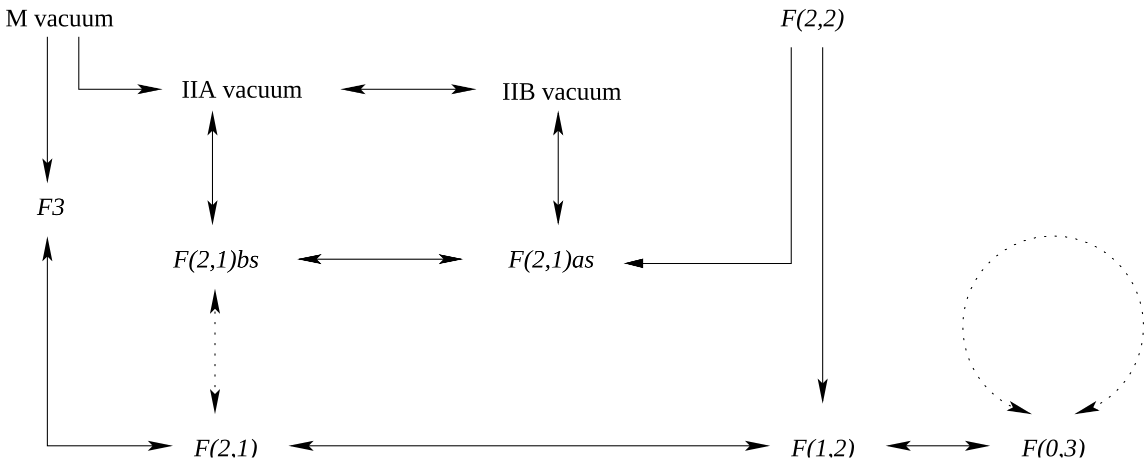

Figure 2: Web of dualities associated with an F3-brane. A dotted line represents an S-duality transformation, a unidirectional solid line represents a Kaluza-Klein reduction from M-theory, and a bidirectional solid line a T-duality transformation.

Using T-duality along the longitudinal $\mathbb{E}^2$ subspace, one generates the same solution in type IIA, namely the $\mathrm{F}(2,1)as$-brane. Both are the T-duals of the corresponding IIA/B vacuum when applying a T-duality transformation along the twisted direction.

Finally, both the $\mathrm{F}(1,2)$-brane and the $\mathrm{F}(2,1)as$-brane can be understood as the Kaluza-Klein reduction of the $\mathrm{F}(2,2)$-brane in eleven dimensions

$$\begin{aligned} F_{(4)} &= F \wedge \mathrm{dvol}(\mathbb{E}^2) \\ ds^2_{(11)} &= \Lambda^{1/3} ds^2(\mathbb{E}^{2,1}) + \Lambda^{-2/3} ds^2(\mathbb{E}^2) + \Lambda^{-1/6} h\,. \end{aligned} \tag{4.10}$$

This finishes the web of dualities for the F3-brane, which is summarised in figure 2.

4.4 F1-string dualities

Due to the fact that there exist two inequivalent fluxstring configurations preserving different amount of supersymmetry, we shall introduce some notation that will allow us to discuss their U-duality properties in a joint discussion. In particular, we shall introduce an index $i = 1, 2$ such that all quantities indexed by $i = 1$ will correspond to the fluxstring preserving 1/4 supersymmetry, whereas those indexed by $i = 2$ will describe the 1/8 configuration.

Performing a T-duality transformation along its worldspace direction, one generates the type IIB $\mathrm{F}(0,1)_i$-string

$$\begin{aligned} \phi_i &= \frac{1}{2}\log\Lambda_i\,, \qquad G^i_{(3)} = F^i \wedge dx \\ g_i &= -\Lambda_i^{1/2} dt^2 + \Lambda_i^{-1/2} dx^2 + h_i\,, \end{aligned} \tag{4.11}$$

where Λ_1 is defined in (3.26), F^1 is the RR 2-form field strength derived from (3.27) and

h_1 is the transverse metric

$$\begin{aligned} h_1 = \Lambda_1^{1/2} \Big[& dr^2 + r^2 d\theta^2 + r^2 \sin^2(2\theta) d\psi^2 + \\ & + d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2 + \tilde{r}^2 \sin^2(2\tilde{\theta}) d\tilde{\psi}^2 \Big] + \\ + \Lambda_1^{-1/2} \Big[& r^2 \left(d\varphi + \cos 2\theta d\psi\right)^2 (1 + \tilde{\beta}\tilde{r}^2) + \tilde{r}^2 \left(d\tilde{\varphi} + \cos 2\tilde{\theta} d\tilde{\psi}\right)^2 (1 + \beta r^2) - \\ & - 2\beta\tilde{\beta} r^2 \tilde{r}^2 \left(d\varphi + \cos 2\theta d\psi\right) \left(d\tilde{\varphi} + \cos 2\tilde{\theta} d\tilde{\psi}\right) \Big] . \end{aligned} \tag{4.12}$$

Similarly, Λ_2 is defined in (3.32), F^2 is the RR 2-form field strength derived from (3.33) and h_2 is the transverse metric

$$\begin{aligned} h_2 = \Lambda_2^{1/2} \Bigg[& dr^2 + r^2 \left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2\right) + \\ & + r^2 \Bigg(\cos^2\theta_1 d\varphi_1^2 + \sin^2\theta_1 \cos^2\theta_2 d\varphi_2^2 + \sin^2\theta_1 \sin^2\theta_2 \cos^2\theta_3 d\varphi_3^2 + \\ & + \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 d\varphi_4^2 \Bigg) \Bigg] - \\ - \Lambda_2^{-1/2} r^4 \Big[& \beta_1 \cos^2\theta_1 d\varphi_1 + \beta_2 \sin^2\theta_1 \cos^2\theta_2 d\varphi_2 + \beta_3 \sin^2\theta_1 \sin^2\theta_2 \cos^2\theta_3 d\varphi_3 + \\ & + \beta_4 \sin^2\theta_1 \sin^2\theta_2 \sin^2\theta_3 d\varphi_4 \Big]^2 . \end{aligned} \tag{4.13}$$

Close to the core, that is, for $r^2\beta_i^2 \ll 1$ for all i, where the supergravity description is reliable since the string coupling constant is weak, (4.11) is flat, whereas far away from the core, that is, for $r^2\beta_i^2 \gg 1$ for all i, it is no longer conformal to a cylindrical metric. It is, in any case, not appropriate to use the supergravity description in that region since the string coupling constant blows up there and one should take into account higher order corrections in g_s into the classical supergravity equations of motion or use its S-dual description, which is the $\mathrm{F}(0,1)_i bs$-string:

$$\begin{aligned} \phi_i &= -\frac{1}{2} \log \Lambda_i , \qquad H_i = -F^i \wedge dx \\ g &= -dt^2 + \Lambda_i^{-1} dx^2 + \Lambda_i^{-1/2} h_i . \end{aligned} \tag{4.14}$$

Notice that the above solution is equivalent to the type IIB configuration obtained by applying a T-duality transformation along the twisted direction on the type IIA vacuum with topological non-trivial identifications.

Of course, one could have started from the flat background in type IIB with non-trivial identifications and generate the $\mathrm{F}(0,1)_i as$-string , which is formally equivalent to (4.14).

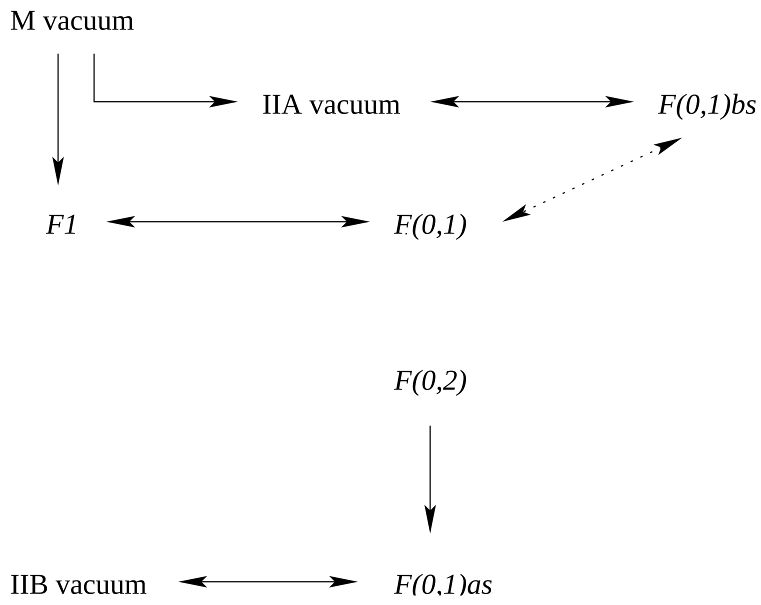

Figure 3: Web of dualities associated with a F1-string. A bold line represents an S-duality transformation, while unidirectional lines indicate Kaluza-Klein reductions from M-theory, and bidirectional lines a T-duality transformation. This diagram is valid for both types of fluxstrings, adding the relevant subscript (1 or 2, but always the same) to the $F(0,1)$ solutions.

This new type IIA solution can be seen as the Kaluza-Klein reduction of the F$(0,2)$-brane

$$\begin{aligned} F^i_{(4)} &= -F^i \wedge \mathrm{dvol}(\mathbb{E}^2) \\ ds^2_{11} &= -\Lambda_i^{1/3} dt^2 + \Lambda_i^{-2/3} ds^2(\mathbb{E}^2) + \Lambda_i^{1/6} h_i \,. \end{aligned} \tag{4.15}$$

This finishes the web of dualities obtained from the F1-string and which is summarised in figure 3.

4.5 N7-brane dualities

The purpose of this subsection is to give a preliminary analysis of the family of solutions related to the N7-brane (3.37). It will be convenient to introduce the following notation

$$h = \Lambda^{1/2} \left(2dx^+ dx^- - (x^1)^2 (dx^-)^2\right) + \Lambda^{-1/2} \left(dx^1 + x^1 x^- dx^-\right)^2 \,, \tag{4.16}$$

where Λ is the one appearing in (3.35).

By T-duality along $7-p$ of the spacelike directions in $\mathbb{E}^7$ in (3.37), one generates a family of N$(p, 7-p)$-branes with dilaton

$$\phi = \frac{p-4}{4} \log \Lambda \,, \tag{4.17}$$

RR field strength

$$F_{(9-p)} = F \wedge \mathrm{dvol}(\mathbb{E}^{7-p}) \,, \tag{4.18}$$

and metric

$$g = \Lambda^{1/2} ds^2(\mathbb{E}^p) + \Lambda^{-1/2} ds^2(\mathbb{E}^{7-p}) + h \,, \tag{4.19}$$

where F is the one defined in (3.37).

One can then act with S-duality on the new type IIB configurations ($p = 6, 4, 2, 0$). The N(4,3)-brane is S-selfdual (hence it has constant dilaton) whereas the N(6,1)-brane is S-dual to the N$(6, 1)bs$-brane

$$\phi = -\frac{1}{2} \log \Lambda \qquad \text{and} \qquad H = -F \wedge dx^7 \,, \tag{4.20}$$

with metric

$$g = ds^2(\mathbb{E}^6) + \Lambda^{-1} d(y^7)^2 + \Lambda^{-1/2} h \,. \tag{4.21}$$

Similarly, the N(2,5)-brane is S-dual to the N$(2, 5)bs$-brane

$$\phi = \frac{1}{2} \log \Lambda \qquad \text{and} \qquad H = 2dx^- \wedge \mathrm{dvol}(\mathbb{E}^2) \,, \tag{4.22}$$

with metric

$$g = \Lambda ds^2(\mathbb{E}^2) + ds^2(\mathbb{E}^5) + \Lambda^{1/2} h \,, \tag{4.23}$$

whereas the N(0,7)-brane is S-dual to the N$(0, 7)bs$-brane

$$\phi = \log \Lambda \qquad \text{and} \qquad F_{(1)} = 2dx^- \,, \tag{4.24}$$

with metric

$$g = \Lambda^{1/2} ds^2(\mathbb{E}^7) + \Lambda h \,. \tag{4.25}$$

One can proceed systematically, uplifting the new type IIA configurations obtained on the process to get new M-theory configurations such as the N(6,2)-brane described by

$$F_{(4)} = F \wedge \mathrm{dvol}(\mathbb{E}^2) \,, \tag{4.26}$$

and metric

$$g = \Lambda^{1/3} ds^2(\mathbb{E}^6) + \Lambda^{-2/3} ds^2(\mathbb{E}^2) + \Lambda^{-1/6} h \,. \tag{4.27}$$

There are many more configurations that go beyond the scope of our present analysis, but deserve further study since, among many other interests, they might lead to more general ansätze to solve the eleven- and ten-dimensional supergravity equations of motion.

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